Quantification of Educational Inequality through the Application of Gini Coefficient in Educational Indices

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Abstract

Education inevitably plays a significant role in the development of a country. Most studies assessing the level of education either quantitatively or qualitatively have dealt with the average of the state-wide level of education. Though, just as important as the average level is the inequality of education for it is likely to transcend directly to the economic inequality of a society. The objective of this paper is to demonstrate the means and importance of observing inequality of educational level within a country. The paper adopts a method proposed by Hojo (2009) and Thomas, Wang, and Fan (2001), Gini coefficient of education. Using this method, it quantified the inequality level of educational attainment and observed its historical progression in Japan, Korea, and the US. Then a comparison was made between the Gini index and the mean years of schooling in order to examine the difference in their progression. It differs from previous papers in that it did not only argue the importance of focusing on educational inequality but also discussed its nature and relation to another more prevalent educational index. Furthermore, it also compared the results of the Gini coefficient in order to assess its validity. As a result, it found that firstly, whilst mean years of schooling increased monotonically in all three countries, the progression of educational inequality varied. Secondly, it also found that there was a large discrepancy in the Gini coefficient of Korea and the US, despite both countries having the majority of their population concentrated at a certain educational level. There are two important implications found from our study. Firstly, it found that increase in the population’s acquired years of schooling does not always mitigate inequality, but in fact, could also lead to a rise in the inequality level. An improved result in mean years of schooling could also reflect a situation where the schooling years were accumulated at tertiary education for a limited portion of the population. Secondly, it also found that such case is more viable in societies with a higher level of mean years of schooling. There are two main directions to be explored in future research. First, the enlargement of the sample geographically and chronologically; second, the identification of the conditions in which application of the Gini coefficient is appropriate.

Keywords: Educational Inequality, Gini Coefficient, Economics of Education, Educational Development, Educational Attainment

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INTRODUCTION

The relation between a country’s social and economic development and its educational environment has been discussed by many (Kurosaki and Khan 2006; Venkataraman 2009; Coombs and Ahmed 1974). In such context, quantification of the educational level of a country is an important task, as it provides a basis for analysis. Nevertheless, it is also a relatively new area of study which is still left with substantial room for elaboration. In particular, there is still much need for methodological refinement for it to be used as a pragmatic tool. In other words, how one should measure collective level of educational attainment and scale it to be viable for comparison between different countries and societies, is an important question to be dealt with. Currently, the most widely used index that serves this role is the mean years of schooling, particularly in empirical analyses (Wail, Hanchane, and Kamal 2011). However, a “mean” merely serves as an estimate of the prevalence of education, and does not account for the distribution within the designated population. Most importantly, it is not able to depict the inequality that is present in the group. In this respect, papers by Hojo (2009) and Thomas et al. (2001) have proposed Gini coefficient as a feasible means by which to account for this aspect of educational attainment.

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The United Nations Development Programme (UNDP)’s Inequality-adjusted Human Development Index (IHDI) introduced in 2010 (UNDP, 2010), also began to measure educational inequality using the Atkinson index. Despite this emerging interest in the educational inequality, a comparison of mean years of schooling and Gini coefficient rests relatively unexplored. When one argues the importance of looking at inequality, the assumption is that its progression does not always occur adjacent to the average level. In order to discuss the usability and the function of this index, it is necessary that this assumption is tested for. In particular, an analogy can be drawn from the theoretical establishment of development economics, where the initial Trickle Down Theory (Aghiton and Bolton 1997) was counter-argued under the claim that economic growth does not directly lead to eradication of poverty. Our interest is to examine if this claim holds true for education; whether the improvement in the population’s acquired years of schooling leads to mitigation of educational inequality.

The purpose of this paper is to examine the progression of educational inequality by making a comparison to mean years of schooling. It will discuss how inequality progresses in a society and how its change differs from that of the average level of education. In order to observe a long-term trend, it will use a dataset dating back to the pre-war period to observe the two indices under different macro trends.

Following this introduction, Section 2 will present the methodology of this study. Section 3 will present the result of the trend of the two indices and discuss the implications that can be drawn out of it. Finally, Section 4 will make the concluding remarks.

### METHODOLOGY

#### Data Source

Among few datasets on education stock, there are useful data such as that of Barro and Lee (2013) offering the mean years of schooling of 146 countries over 50 years. Despite the fact that the paper is frequently referred by their co-researchers, there is an important limitation to this data whose estimate is calculated from completed level of education and thus does not reflect years of schooling attained by those who dropped out or repeated years. Consequently, mean years of schooling are underestimated in their dataset. Such limitation of this dataset has also been pointed out by previous studies (Godo 2011). In response to this limitation of the dataset by Barro and Lee (2013), Godo (2011) has provided an estimated dataset with an improved accuracy and detail. This study will use this estimated data provided by Godo (2011).

Although the dataset covers only 3 countries, the estimate is calculated from the actual enrolment and population data both of which are objectively measured, making it more accurate than an estimated value which is calculated from completed level of education. Furthermore, the time span covered by this dataset extends to 80 years, dating back to 1920 until 2000. Since the dataset with greater accuracy and longer time span is preferred when examining the behaviour of indices, this study uses the estimated data of Godo (2011) as its data source. The data in Godo (2011) is presented in Table 1. In order to make the educational level comparable, the designated full years of schooling was framed according to the US’s educational system, in which 1st to 8th grades is primary education, 9th to 12th grades is secondary education, and everything over 12th grade is tertiary education.

<table>
<thead>
<tr>
<th>Year</th>
<th>Japan</th>
<th>Korea</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Primary</td>
<td>Secondary</td>
<td>Tertiary</td>
</tr>
<tr>
<td>1920</td>
<td>4.1</td>
<td>0.17</td>
<td>0.022</td>
</tr>
<tr>
<td>1930</td>
<td>5.1</td>
<td>0.40</td>
<td>0.073</td>
</tr>
<tr>
<td>1940</td>
<td>5.7</td>
<td>0.69</td>
<td>0.12</td>
</tr>
<tr>
<td>1950</td>
<td>6.3</td>
<td>1.1</td>
<td>0.17</td>
</tr>
<tr>
<td>1960</td>
<td>6.9</td>
<td>1.6</td>
<td>0.24</td>
</tr>
<tr>
<td>1970</td>
<td>7.3</td>
<td>2.1</td>
<td>0.37</td>
</tr>
<tr>
<td>1980</td>
<td>7.6</td>
<td>2.6</td>
<td>0.58</td>
</tr>
<tr>
<td>1990</td>
<td>7.8</td>
<td>3.0</td>
<td>0.75</td>
</tr>
<tr>
<td>2000</td>
<td>8.0</td>
<td>3.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Source: Based on Godo (2011)
Gini Coefficient of Education

This study uses Gini coefficient of education as an index for educational inequality. If we follow a geometrical interpretation, Gini coefficient is determined by the Lorentz curve, which plots the percentage of the total schooling years of the population on vertical axis that is cumulatively earned by the bottom X% of the population. The graph demonstrates what percentage of the total schooling years of population is distributed to the bottom X% of population. Therefore, the equidistribution line represents the perfectly equal distribution of the total schooling years. Gini coefficient of education is the ratio of the area between the equidistribution line and the Lorentz curve to the area under the equidistribution line. As a consequence of definitions, Gini coefficient falls in the range between 0 and 1, and the smaller Gini coefficient is, more equal distribution of acquired years of schooling is in the population. Algebraically, the index can be defined as follows:

\[
\text{Educ.Gini} = 1 - 2 \int_0^1 L(x)\,dx
\]

\[Y = L(x)\text{ is Lorentz curve}\]

Processing of the Dataset

Godo (2011) offers the estimated datasets of mean schooling years by levels of education, but since Lorentz curve is acquired from the percentage of cumulative population, Gini coefficient cannot be calculated without this data. In order to extract a cumulative population, it is assumed that at primary, secondary, and tertiary education respectively, the individual had either fully completed or had not enrolled at all. The population is then allocated into either case according to the value of mean years of schooling. For example, if the mean schooling years of elementary education is \(\alpha\) years, that of secondary education is \(\beta\) years, and that of tertiary education is \(\gamma\) years, we assume that 0 year of schooling is distributed to the individuals in \((1 - \frac{\alpha}{2})\) of total population, 8 years of schooling are distributed to the individuals in \((\frac{\alpha}{2} - \frac{\beta}{2})\) of total population, 12 years of schooling are distributed to the individuals in \((\frac{\beta}{4} - \frac{\gamma}{4})\) of total population, and 16 years of schooling are distributed to the individuals in \((\frac{\gamma}{2})\) of total population. Through this manipulation, Lorentz curve is drawn without distortion in the estimated value of mean schooling years by level of education. See Appendix A for the Proof. In our study, designated full years of schooling for tertiary education is defined as 4 years. Although in reality, it is necessary to consider graduate school, this assumption does not affect the result significantly as the maximum value was 2.2 years for the US in 2000. Since estimate in Godo (2011) includes extra schooling years attained by students who repeated years, the mean years of schooling in primary level has exceeded 8 years in the US between 1940 and 2000. In such case, one must be mindful of the fact that the value of the population ratio allocated to 0 year of schooling becomes negative when it is processed in the same manner as the other years. Because the population is allocated either to 0 or 8 years of schooling according to the mean value, the population ratio of those who attained 0 year, \((1 - \frac{\alpha}{8})\), takes a negative value when mean years of schooling in primary level exceeds 8 years. As it is impossible for this value to be negative under the empirical assumption, the population exceeding 8 years was shifted to secondary education. This additional data process was adopted as it does not distort the overall mean years of schooling. The processed data are presented in Table 2. The population ratio by attained levels of education is presented in Table 3.

Table 2: Processed mean years of schooling by levels of education: Japan, Korea, and the US

<table>
<thead>
<tr>
<th>Year</th>
<th>Japan Primary</th>
<th>Japan Secondary</th>
<th>Japan Tertiary</th>
<th>Korea Primary</th>
<th>Korea Secondary</th>
<th>Korea Tertiary</th>
<th>US Primary</th>
<th>US Secondary</th>
<th>US Tertiary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920</td>
<td>4.1</td>
<td>0.17</td>
<td>0.022</td>
<td>0.63</td>
<td>0.0012</td>
<td>0.0003</td>
<td>7.7</td>
<td>0.42</td>
<td>0.18</td>
</tr>
<tr>
<td>1930</td>
<td>5.1</td>
<td>0.40</td>
<td>0.073</td>
<td>0.80</td>
<td>0.0075</td>
<td>0.0011</td>
<td>8.0</td>
<td>0.80</td>
<td>0.26</td>
</tr>
<tr>
<td>1940</td>
<td>5.7</td>
<td>0.69</td>
<td>0.12</td>
<td>1.1</td>
<td>0.022</td>
<td>0.0028</td>
<td>8.0</td>
<td>1.5</td>
<td>0.33</td>
</tr>
<tr>
<td>1950</td>
<td>6.3</td>
<td>1.1</td>
<td>0.17</td>
<td>2.1</td>
<td>0.16</td>
<td>0.028</td>
<td>8.0</td>
<td>2.1</td>
<td>0.45</td>
</tr>
<tr>
<td>1960</td>
<td>6.9</td>
<td>1.6</td>
<td>0.24</td>
<td>2.8</td>
<td>0.34</td>
<td>0.071</td>
<td>8.0</td>
<td>2.7</td>
<td>0.61</td>
</tr>
<tr>
<td>1970</td>
<td>7.3</td>
<td>2.1</td>
<td>0.37</td>
<td>3.9</td>
<td>0.64</td>
<td>0.14</td>
<td>8.0</td>
<td>3.1</td>
<td>0.89</td>
</tr>
<tr>
<td>1980</td>
<td>7.6</td>
<td>2.6</td>
<td>0.58</td>
<td>5.4</td>
<td>1.2</td>
<td>0.20</td>
<td>8.0</td>
<td>3.4</td>
<td>1.4</td>
</tr>
<tr>
<td>1990</td>
<td>7.8</td>
<td>3.0</td>
<td>0.75</td>
<td>6.5</td>
<td>1.9</td>
<td>0.47</td>
<td>8.0</td>
<td>3.6</td>
<td>1.8</td>
</tr>
<tr>
<td>2000</td>
<td>8.0</td>
<td>3.3</td>
<td>1.0</td>
<td>7.1</td>
<td>2.5</td>
<td>0.86</td>
<td>8.0</td>
<td>3.7</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Source: Based on Godo (2011)
**RESULTS AND DISCUSSION**

**Mean Years of Schooling in Japan, Korea, and the US**

The first subsection will examine the results of mean years of schooling in Japan, Korea, and the US. Table 4 shows the progression of the index in the three countries between 1920 and 2000. In all three countries, the mean years have increased monotonically. As for Korea, whilst the result is less than a year until 1930, it surpasses 10 years in 2000. On the other hand, in the US where the result was already 8.3 years at 1920, the change is not as radical as it marks 14.0 years at 2000. This trend agrees with those presented in previous studies such as Barro and Lee (2013) and Ministry of Education, Culture, Sports, and Technology, Japan (2005). According to the datasets of Barro and Lee (2013), the value of mean years of schooling of Korea which was 5.02 in 1955 declined to 4.20 in 1960. However, since this is the only exception to the monotonic trend in the datasets, its overall trend agrees with our result. Godo (2011) also pointed out the possibility that this figure might be a statistical error as this is the only exception to the trend.

**Table 4: Mean years of schooling: Japan, Korea, and the US**

<table>
<thead>
<tr>
<th>Year</th>
<th>Japan</th>
<th>Korea</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920</td>
<td>4.3</td>
<td>0.6</td>
<td>8.3</td>
</tr>
<tr>
<td>1930</td>
<td>5.6</td>
<td>0.8</td>
<td>9.1</td>
</tr>
<tr>
<td>1940</td>
<td>6.5</td>
<td>1.1</td>
<td>9.8</td>
</tr>
<tr>
<td>1950</td>
<td>7.6</td>
<td>2.3</td>
<td>10.5</td>
</tr>
<tr>
<td>1960</td>
<td>8.7</td>
<td>3.3</td>
<td>11.3</td>
</tr>
<tr>
<td>1970</td>
<td>9.8</td>
<td>4.8</td>
<td>12.0</td>
</tr>
<tr>
<td>1980</td>
<td>10.7</td>
<td>6.9</td>
<td>12.8</td>
</tr>
<tr>
<td>1990</td>
<td>11.5</td>
<td>9.0</td>
<td>13.5</td>
</tr>
<tr>
<td>2000</td>
<td>12.3</td>
<td>10.5</td>
<td>14.0</td>
</tr>
</tbody>
</table>

Source: Based on Godo (2011)
Educational Gini Coefficient in Japan, Korea, and the US

Table 5 and Figure 2 show the historical progression of the Gini coefficient of the three countries over 80 years dating back to pre-war (1920-2000). This demonstrates that in both Japan and Korea, there has been an extremely rapid decline in the inequality level. This compares to the US whose level has stayed largely the same. Moreover, there are even points at which there is a slight increase in its value. These trends agree with the results of Thomas et al. (2001) and Ziesemer (2016). Since both of Thomas et al. (2001) and Ziesemer (2016) are based on the datasets of Barro and Lee (2013), the figure of Korea in 1965 is the only exception to the trend.

Table 5: Gini coefficient of education: Japan, Korea, and the US

<table>
<thead>
<tr>
<th>Year</th>
<th>Japan</th>
<th>Korea</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920</td>
<td>0.51</td>
<td>0.92</td>
<td>0.10</td>
</tr>
<tr>
<td>1930</td>
<td>0.41</td>
<td>0.90</td>
<td>0.097</td>
</tr>
<tr>
<td>1940</td>
<td>0.36</td>
<td>0.87</td>
<td>0.085</td>
</tr>
<tr>
<td>1950</td>
<td>0.30</td>
<td>0.76</td>
<td>0.091</td>
</tr>
<tr>
<td>1960</td>
<td>0.24</td>
<td>0.69</td>
<td>0.086</td>
</tr>
<tr>
<td>1970</td>
<td>0.20</td>
<td>0.57</td>
<td>0.096</td>
</tr>
<tr>
<td>1980</td>
<td>0.17</td>
<td>0.41</td>
<td>0.10</td>
</tr>
<tr>
<td>1990</td>
<td>0.13</td>
<td>0.30</td>
<td>0.097</td>
</tr>
<tr>
<td>2000</td>
<td>0.11</td>
<td>0.23</td>
<td>0.090</td>
</tr>
</tbody>
</table>

Source: Calculated based on Table 3

Figure 2. Gini coefficient of education: Japan, Korea, and the US

Source: Based on Table 5

Comparison

The result presented in Table 4 and 5 shows that the relation between mean years of schooling and educational Gini coefficient is not uniform in the three case study countries. In fact, it has proved that the latter does not progress adjacent to the former. In the case of Japan and Korea, both mean years of schooling and Gini coefficient of education have demonstrated gradual improvement, signifying that the decrease in the inequality of educational attainment has occurred simultaneously with the increase in the population’s acquired years of schooling. On the other hand, in the US, the results between 1960 and 1980 demonstrate that whilst the population had acquired increased years of schooling, there was, in fact, an increase in the inequality level.

Firstly, this implies that the increase in the population’s schooling years does not necessarily occur from the lower level. In other words, the increase in mean years of schooling does not always signify that there are more children enrolled for primary education. It could just as well reflect a situation where the population which had completed secondary education proceeded to tertiary education. As mentioned by Holsinger (2005), a country may invest disproportionately in tertiary education, consequently skewing the distribution of educational attainment. If the population attaining primary school education rests unchanged, such progress would, in fact, lead to an enlarged inequality in educational attainment in that society. In case of the US, after primary education became
prevalent, the growth rate of tertiary education was higher than that of secondary education which had led to this widening inequality.

The second implication the results have is that the level of the mean value itself could affect how it relates to Gini coefficient of education. Looking at the progression of mean years of schooling in Japan and Korea, one is able to see that their attainment level increases substantially. Whereas the mean value is 4.3 years in Japan, and 0.6 years in Korea in 1920, the level soars up to 12.3 years and 10.5 years in 2000. In contrast, in the US, the mean level already reaches 8.3 years in 1920 which increases to 14.0 years by 2000. In both Japan and Korea, access to education was still very limited in 1920. The mean value is both below 6 years, signifying that an average person had not finished primary education. The high level of inequality in these countries reflects a society where the majority of the population has barely had basic level of education. This observed trend agrees with that which has been pointed out for other underdeveloped societies. For example, Mesa (2007) indicated that educational Gini index and mean years of schooling of regions and provinces in the Philippines are negatively associated with each other. Holsinger (2005) also showed that Vietnam successfully increased its educational attainment whilst also mitigating educational inequality. On the other hand, in the US, where the basic level of education was already prevalent, the increase of the schooling years began to occur at different levels in which its effect on the two variables was not as tangible as in the case of Japan and Korea.

There are also several limitations to the study which should be pointed out. Firstly, this study has found that there is a limitation in the Gini coefficient itself, which is mentioned in previous studies (Mellor 1989; Bellu and Liberati 2006; Kwok 2010). Although Gini coefficient has been the most popular method (De Maio 2007) to quantify inequality due to its mathematical nature, its estimate of inequality is liable to the level at which this inequality occurs (Yildiz and Kayili 2015). The comparison between the Gini index of Korea and the US is a good example to demonstrate this. As mentioned in the Result and Discussion, when we calculate the mean years of schooling at each educational level, it is 0.63 years in primary education and 0.0012 years in secondary education for Korea in 1920, signifying that most people had not completed primary education. On the other hand in the US, the mean value was 7.7 years in primary education and 0.42 years in secondary education in 1920, meaning that an average person’s educational attainment was slightly below completion of primary education. The Gini index is 0.92 for Korea, and 0.10 for the US, representing a large difference in the inequality level between the two countries.

However, one must be cautious that Gini coefficient is an index that serves to demonstrate the degree of disparity irrespective of the average level. In other words, it should not question at which level the society is equal, it should only observe the distribution. Korea in 1920 may be limited at the average level of educational attainment. However, if the majority of the population had not finished primary education, it could also be described as largely equal, just as much as the case of the US in which the majority of the population had completed primary school but had not been to secondary school. It has, therefore, been revealed through the results that Gini coefficient is affected by the absolute level of educational attainment, and thus does not always reflect inequality. Consequently, inequality is overestimated in societies where there is a high representation of population in the lowest percentile.

Secondly, the number of countries and the timespan are also limitations of our study. This paper only analysed Japan, Korea, and the US, and its data only covered up to 2000. In future studies, we wish to enlarge our scope both geographically and chronologically in order to apply our findings to more diverse samples to validate the claim.

CONCLUSION

This paper discussed how the progress of inequality in educational attainment differs from that of the prevalence of education. It differs from previous papers in that it did not only argue the importance of focusing on educational inequality but also discussed its nature and relation to other more prevalent educational indices. Firstly, it found that inequality is not always mitigated as a result of the prevalence of education, but in fact, an increased prevalence could lead to a rise in inequality. Secondly, it was also implied that such case is more viable in societies with higher level of mean years of schooling. Thirdly, it has also made a methodological finding that there is a limitation of Gini coefficient in depicting inequality for its tendency to overestimate the inequality level in a society where there is a high representation of the lowest percentile. This paper has also
made a methodological contribution by proposing a data processing method to subtract the size of population of the respective educational level from limited information. The method is also useful in that it does not distort the mean years of schooling when doing so. This has opened a path for quantitative analysis in many more countries where raw data on education are often scarce.

The empirical analysis of our paper demonstrates that there is importance of observing the educational inequality when discussing the educational attainment as part of social development. The variable has the ability to present an aspect that is not necessarily portrayed by the other prevalent indices of the domain, especially when comparing societies at different development levels. Though it should also be kept in mind, how and when this index is applied for its aforementioned limitation. In future work, first of all, we will enlarge our sample both geographically and chronologically in order to verify the index with further accuracy. We will also seek to find more specific conditions in which the application of educational Gini coefficient is appropriate to contribute to further refinement of methodological development.

REFERENCES


Appendix A

Assertion
Allocating population through the method mentioned in paper doesn’t distort mean years of schooling by levels of education.

Proof
Let
\[ X = \text{Population}; \]
\[ A \text{ years} = \text{Desinated full years of schooling for primary education}; \]
\[ B \text{ years} = \text{Desinated full years of schooling for secondary education}; \]
\[ C \text{ years} = \text{Desinated full years of schooling for tertiary education}; \]
\[ \alpha_a \text{ years} = \text{Mean years of schooling in primary education}; \]
\[ \beta_a \text{ years} = \text{Mean years of schooling in secondary education}; \]
\[ \gamma_a \text{ years} = \text{Mean years of schooling in tertiary education}. \]

Now allocated population by attained years of schooling is calculated as follows:
\[
\begin{align*}
0 \text{ year} & \quad X \left(1 - \frac{\alpha_a}{A}\right) \\
A \text{ years} & \quad X \left(\frac{\alpha_a}{A} - \frac{\beta_a}{B}\right) \\
B \text{ years} & \quad X \left(\frac{\beta_a}{B} - \frac{\gamma_a}{C}\right) \\
C \text{ years} & \quad X \left(\frac{\gamma_a}{C}\right)
\end{align*}
\]

Let
\[ \alpha_e \text{ years} = \text{Calculated mean years of schooling in primary education based on allocated population} \]
\[ \beta_e \text{ years} = \text{Calculated mean years of schooling in secondary education based on allocated population} \]
\[ \gamma_e \text{ years} = \text{Calculated mean years of schooling in tertiary education based on allocated population} \]

Now \( \alpha_e \) is calculated as follows:
\[
\alpha_e = \frac{0 \cdot X \left(1 - \frac{\alpha_a}{A}\right) + A \cdot X \left(\frac{\alpha_a}{A} - \frac{\beta_a}{B}\right) + B \cdot X \left(\frac{\beta_a}{B} - \frac{\gamma_a}{C}\right) + C \cdot X \left(\frac{\gamma_a}{C}\right)}{X} = \alpha_a
\]

In the same way,
\[ \beta_e = \beta_a \] and
\[ \gamma_e = \gamma_a \]

From the above, the assertion is proven.