A tournament scheduling model for National Basketball Association (NBA) regular season games

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Abstract

Professional sports have become a business market with high rewards in global economies. In particular the high level leagues can attract a group of sport populations over global countries, such as the National Basketball Association (NBA). Tournaments are a main part for the whole operation of the professional sport leagues that can maintain and increase the attractiveness to fans. The NBA currently consists of 30 teams homed at different cities in USA and Canada. The whole tournament of its regular season every year lasts to play nearly 24 weeks. Nevertheless, the tournament scheduling problem is complicated and large-scale and subject to many factors, such as dates, venues, opponents of games, etc. This study aims to formulate a mathematical programming model for the tournament scheduling of the NBA based on the fairness of opponent arrangement. An Integer Programming (IP) model was proposed to minimize the difference of opponent competitiveness among all teams. Opponent competitiveness is defined as the sums of opponent winning percentages in the last season for every game. Constraints include some known regulations and previous practices in published journalism reports. Numerical experiments revealed this model can obtain promising results for the tournament arrangement of the NBA.

Keywords: Tournament Scheduling, National Basketball Association (NBA), Integer Programming (IP), Opponent Competitiveness

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INTRODUCTION

Nowadays, sports are not just general leisure activities but have also become a business market with high rewards because of the development of various professional athleticisms. In particular a variety of professional sport leagues attract people over the world to be their fans, such as the National Football League (NFL), the Major League Baseball (MLB), and the NBA in the USA. Tournaments of every kind of professional athleticisms can create huge benefits in terms of high ratings of TV relays and large sales of tickets and peripheral products.

Tournament scheduling is one of the important promotional tools for professional leagues. Fans always notice the tournament arrangement of teams of their star athletes. Tournament fierceness and fairness are main discussed topics of team management, athletes and fans because the schedules affect the whole season performance for all teams and all athletes. However, tournament scheduling is not an easy task by manual arrangement for a large league like the NBA with 30 teams, as the schedules have to follow numerous and complicated restrictions. It is not to mention to meet the objectives of fierceness and fairness in such rules. We are motivated to propose a tournament scheduling model with a fair objective for the NBA regular season games. An appropriate schedule arrangement can increase the attractiveness of plays to the fans and even absorb more extensive spectator groups. Furthermore, such arrangement may promote the performance of athletes and reduce their negative impacts on psychological and physiological sides.

Teams in the NBA have their own home courts located at different cities in the USA and Canada. They are divided into various conferences and divisions. Each team plays same and fixed number of total games with even home and away during certain days in a regular season. Games competing with teams across conferences and divisions have a flexible condition. The decision variables are the venues for home/away teams in certain available

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dates with commercial considerations or without conflict activities in the venues. The purpose of this study is to formulate a mathematical programming model for tournament scheduling of the NBA regular season games. Subject to the scheduling rules, an IP model was proposed to minimize the difference of opponent competitiveness among all teams. Through some case tests, this study attempts to ensure that the robustness of solution results can be applied to the practical programming. The remaining contents of this paper make a literature review regarding sport tournament scheduling in Section 2. Section 3 introduces the tournament schedule rules in the NBA and the proposed model. Section 4 discusses the results of some test cases. The final section summarizes our study and provides some suggestions for the future research.

LITERATURE REVIEW

Scheduling problems in sports are required such as the arrangement for the tournaments and umpires. Wright (2009) reviewed the works using Operations Research (OR) techniques for about 50 years. Ribeiro (2012) analyzed the fundamental problems for sport scheduling and provided a survey of applications of optimization methods to scheduling problems in professional leagues of different sport disciplines. The main problems arising in tournament scheduling include issues such as breaks minimization, distance minimization, the Traveling Tournament Problem (TTP), and carry-over effects minimization. These problems have been solved by different exact and approximate approaches, including integer programming, constraint programming, metaheuristics, and hybrid methods. Some efforts are devoted to the specific problems. Costa (1995) combined the mechanisms of genetic algorithms and tabu search to solve National Hockey League (NHL) optimization problems, hoping to reduce costs. This hybrid method developed is well suited for Open Shop Scheduling Problems. Dilkina and Havens (2004) used Constraint Satisfaction Problem (CSP) to schedule 256 games in 17 weeks for 32 teams of the NFL. These teams were divided into the National Football Conference (NFC) and the American Football Conference (AFC). Some home/away spacing constraints and other related regulations required to be adhered. An available schedule had to be used with the US television network live. They provided an overview of the constraint solving methodology employed and the implementation of the NFL prototype system. Duran, Guajardo, and Wolf-Yadlin (2012) proposed an integer linear programming (ILP) model to schedule the Second Division of the Chilean professional soccer league. Geographical restrictions are particularly important because of the special geographical environment of Chile for long bus travel distances. Chilean league officials have successfully used this model to schedule all five Second Division tournaments between 2007 and 2010.

There were studies focused on the nonprofessional basketball games. Nemhauser and Trick (1998) developed a combination of integer programming and enumerative techniques to schedule nine universities in the Atlantic Coast Conference (ACC). A basketball competition assigned schools to play home and road games against each other over a nine-week period. This work was applied to make reasonable schedules in 1997-1998 seasons. Wright (2006) used a variant of simulated Annealing (SA) to schedule games for National Basketball League of New Zealand. This approach was applied to make the schedule in 2004 season.

For the NBA, recent research devoted to various perspectives, such as statistical analysis for winning playoffs (Summers 2013), identifying basketball performance indicators in regular season and playoff games (Garcia et al. 2013), time trends for injuries and illness, and their relation to performance (Podlog et al. 2015). The studies for tournament scheduling are relatively limited. Bean and Birge (1980) constructed schedules of NBA based on Traveling Salesman Problem (TSP). The objective is to reduce the total number of passenger miles for the league travels, 22 teams in the NBA at the time. This approach successfully developed schedules for the 1978-1979 and 1979-1980 seasons with savings of 20.4% costs. Actually, current structure in the NBA has been changed than 20 years ago. More teams and more fans make the tournament scheduling problem become more complicated and commercialized.

METHODOLOGY

This section briefly introduces current teams in the NBA and scheduling rules for their tournaments. A proposed mathematical model can then be formulated with the objective of minimizing the difference of opponent competitiveness among all teams. Opponent competitiveness is defined as the sums of opponent winning percentages in the last season for every game.
Problem Description

The NBA has been developed for nearly 70 years. Initially, there are only 11 teams in NBA, each team playing 60 games in a season. Currently, the NBA has become a huge sport league, with 30 teams homed at different cities in USA and Canada. Figure 1 shows the location of home grounds for these teams. They are divided into 2 conferences, eastern conference and western conference, and belong to 6 divisions, respectively.

Eastern Conference

a. Atlantic division includes Boston Celtics, Brooklyn Nets, New York Knicks, Philadelphia 76ers, and Toronto Raptors.
b. Central division includes Chicago Bulls, Cleveland Cavaliers, Detroit Pistons, Indiana Pacers, and Milwaukee Bucks.
c. Southeast division includes Atlanta Hawks, Charlotte Hornets, Miami Heat, Orlando Magic and Washington Wizards.

Western Conference

a. Southwest division includes Dallas Mavericks, Houston Rockets, Memphis Grizzlies, New Orleans Pelicans, and San Antonio Spurs.
b. Northwest division includes Denver Nuggets, Minnesota Timberwolves, Oklahoma City Thunder, Portland Trail Blazers, and Utah Jazz.

Figure 1. Location of home grounds for all NBA teams

Related scheduling regulations in scheduling tournaments in the NBA are collected as follows:

- Each team must play 82 games in a season, 41 home games and 41 road games.
- Each team only has 1 game at most in a day.
- Each team only has 2 games at most within 3 days.
- More teams have to play on Friday and Saturday.
- Lakers and Clippers use the same court, i.e., Staples Center, so they cannot use the home team identity to play game on same day.
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- No games can be played at the Thanksgiving Day, Christmas Eve, All-Star Game and NCAA finals.
- A team has to play road games when its home court cannot be used in a particular period of time.
- Each team has to play 4 games (2 home games and 2 road games) with the other 4 teams in the same division.
- Each team has to play 2 games (1 home game and 1 road game) with the other 15 teams in different conferences.
- Each team has to play 3 or 4 games (1 home game and 2 road game, 2 home game and 1 road game, or 2 home game and 2 road game) with the other 10 teams in the same conference but different divisions.
- Some special games are arranged on special dates, such as a deliberate game for Warriors and Cavaliers (last season’s finals) at Christmas in 2015-16 season.

Mathematical Model

The proposed model aims to minimize the difference of opponent competitiveness among all teams subject to the scheduling regulations mentioned in the methodology Section. Before introducing the contents of the model, notation for this model is defined as follows:

Indices:
- d: Indices for scheduling days
- i, j: Indices for team

Sets:
- \( W \): the set of days for Friday and Saturday.
- \( H \): the set of days that cannot arrange games.
- \( R_i \): the set of days that have to arrange road games for team i.
- \( S \): the set of days that have to arrange particular games.
- \( E_{d} \): the teams must play on day d in.
- \( X \): the set of teams for shared home.

Decision variables:
- \( x_{dij} \): if team i plays home game with team j at day d ground; 1 for yes, 0 otherwise.
- \( Z_{i} \): opponent competitiveness for team i, i.e., the sum of opponent winning percentages in the last season for every game for team i.
- \( Z_{h} \): the highest opponent competitiveness among all teams.
- \( Z_{l} \): the lowest opponent competitiveness among all teams.

Parameters:
- \( C_i \): the conference of team i.
- \( V_i \): the division of team i.
- \( W_j \): the last seasonal winning percentage of team j.
- \( D \): the total days of a regular season.
- \( K \): the total number of teams.
- \( a \): the total number of games for each team playing in a regular season.
- \( f \): the total number of home (away) games with teams of the same conference and the same division.
- \( g \): the total number of home (away) games with teams of the different conference.
- \( h \): upper bound of total number of home (away) games with teams of the same conference but different divisions.
- \( l \): lower bound of total number of home (away) games with teams of the same conference but different divisions.
- \( m \): maximum days for consecutive plays.
- \( n \): upper bound of the consecutive games in maximum days for consecutive plays.
- \( q \): lower bound of total number of home and road games with teams of the same conference but different divisions.
- \( r \): a ratio of teams playing on Friday and Saturday.

As the Equation (1), the objective function of this model is to minimize the difference of opponent competitiveness. It can be calculated by the difference of \( Z_{h} \) and \( Z_{l} \).

\[
\text{Minimize} = Z_{h} - Z_{l} \quad (1)
\]
Some constraints require to be followed. Opponent competitiveness definition constraints include Equation (2) to (4). Equation (2) is the sum of opponent winning percentages in the last season for every game for team i.

\[
Z_i = \sum_d \sum_j w_j x^d_{ij} + \sum_d \sum_j w_j x^d_{ji} \quad \forall i
\]  

\[Z_h \geq Z_i \quad \forall i
\]  

\[Z_l \leq Z_i \quad \forall i
\]  

Equation (5) to (7) are the limitations for games. Equation (5) means that the total number of home games must be equal to road games. Equation (6) enforces that the total number of games of each team must be the same. Equation (7) limits each team to play only one game at most in a day.

\[
\sum_d \sum_j x^d_{ij} = \sum_d \sum_j x^d_{ji} \quad \forall i
\]  

\[
\sum_d \sum_j x^d_{ij} + \sum_d \sum_j x^d_{ji} = a \quad \forall i
\]  

\[
\sum_j x^d_{ij} + \sum_j x^d_{ji} \leq 1 \quad \forall i, d
\]  

Equation (8) can limit each team to play only n games at most in consecutive m days.

\[
\sum_{d' = d}^{d + (m - 1)} \sum_j x^d_{ij} + \sum_{d' = d}^{d + (m - 1)} \sum_j x^d_{ji} \leq n \quad \forall i, d = 1, 2, \ldots, D - (m + 1)
\]  

If two or more teams use same court, they cannot use the home team identity to play game on the same day as Equation (9).

\[
\sum_j x^d_{ij} + \sum_j x^d_{i'j} \leq 1 \quad \forall d, (i, i') \in X
\]

Equation (10) enforces that specific days can play no games. On the other hand, certain Fridays and Saturdays must play games for commercial considerations as Equation (11). Equation (12) can schedule special games on special dates for certain terms. Equation (13) limits a team to play road games when its home court cannot be used in a particular period of time.

\[
\sum_i \sum_j x^d_{ij} + \sum_i \sum_j x^d_{ji} = 0 \quad \forall d \in \bar{H}
\]

\[
\sum_i \sum_j x^d_{ij} \geq \frac{rK}{2} \quad \forall d \in \overline{W} - \bar{H}
\]

\[
x^d_{ij} + x^d_{ji} = 1 \quad \forall d \in \overline{S}, (i, j) \in \overline{E}_d
\]

\[
\sum_j x^d_{ij} = 0 \quad \forall i, d \in \overline{R}_i
\]

Equations (14) to (22) are constraints playing with opponents in conferences and divisions. Equations (14) and (15) consider the total number of home (away) games for same conference same division. Equations (16) to (17) limit the total number of home (away) games for different conferences. Equations (18) to (22) limit the total number of home (away) games for same conference but different divisions. Equations (23) and (24) are nature of the variables.
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\[ \sum_d x_{ji}^d = f \quad \forall i, j \neq i (V_i = V_j) \]  \hspace{1cm} (15)

\[ \sum_d x_{ij}^d = g \quad \forall i, j \neq i (C_i \neq C_j) \]  \hspace{1cm} (16)

\[ \sum_d x_{ji}^d = g \quad \forall i, j \neq i (C_i \neq C_j) \]  \hspace{1cm} (17)

\[ \sum_d x_{ij}^d + \sum_d x_{ji}^d \geq q \quad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \]  \hspace{1cm} (18)

\[ \sum_d x_{ij}^d \leq h \quad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \]  \hspace{1cm} (19)

\[ \sum_d x_{ij}^d \geq l \quad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \]  \hspace{1cm} (20)

\[ \sum_d x_{ji}^d \leq h \quad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \]  \hspace{1cm} (21)

\[ \sum_d x_{ji}^d \geq l \quad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \]  \hspace{1cm} (22)

\[ x_{ij}^d \in \{0, 1\} \]  \hspace{1cm} (23)

\[ Z_h, Z_l, Z_i \geq 0 \]  \hspace{1cm} (24)

**NUMERICAL EXPERIMENT**

The proposed model is an integer programming problem. This section presents a verified case with smaller problem scale for discussion in detail. Then, the solution results for the real-world case of 2015-2016 regular season tournaments are reported.

**Verified Case**

The test case only selects 12 teams, as shown in Table 1, for tournaments scheduling to play 30 games each team within 65 days. Detailed settings of input data for scheduling regulations are presented in Table 2.

<table>
<thead>
<tr>
<th>Team</th>
<th>Location</th>
<th>Winning Percentages (last season)</th>
<th>Conference</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Celtics</td>
<td>Boston</td>
<td>0.488</td>
<td>Eastern</td>
<td>Atlantic</td>
</tr>
<tr>
<td>Nets</td>
<td>Brooklyn</td>
<td>0.463</td>
<td>Eastern</td>
<td>Atlantic</td>
</tr>
<tr>
<td>Bulls</td>
<td>Chicago</td>
<td>0.610</td>
<td>Eastern</td>
<td>Central</td>
</tr>
<tr>
<td>Cavaliers</td>
<td>Cleveland</td>
<td>0.646</td>
<td>Eastern</td>
<td>Central</td>
</tr>
<tr>
<td>Hawks</td>
<td>Atlanta</td>
<td>0.732</td>
<td>Eastern</td>
<td>Southeast</td>
</tr>
<tr>
<td>Hornets</td>
<td>Charlotte</td>
<td>0.402</td>
<td>Eastern</td>
<td>Southeast</td>
</tr>
<tr>
<td>Mavericks</td>
<td>Dallas</td>
<td>0.610</td>
<td>Western</td>
<td>Southwest</td>
</tr>
<tr>
<td>Rockets</td>
<td>Houston</td>
<td>0.683</td>
<td>Western</td>
<td>Southwest</td>
</tr>
<tr>
<td>Nuggets</td>
<td>Denver</td>
<td>0.366</td>
<td>Western</td>
<td>Northwest</td>
</tr>
<tr>
<td>Timberwolves</td>
<td>Minnesota</td>
<td>0.195</td>
<td>Western</td>
<td>Northwest</td>
</tr>
<tr>
<td>Clippers</td>
<td>Los Angeles</td>
<td>0.683</td>
<td>Western</td>
<td>Pacific</td>
</tr>
<tr>
<td>Lakers</td>
<td>Los Angeles</td>
<td>0.256</td>
<td>Western</td>
<td>Pacific</td>
</tr>
</tbody>
</table>
Table 2: Settings of input data for the test case

<table>
<thead>
<tr>
<th>Regulations</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>The total number of home (road) games for same conference same division</td>
<td>2</td>
</tr>
<tr>
<td>The total number of home (road) games for different conferences</td>
<td>1</td>
</tr>
<tr>
<td>Upper bound of total number of home (road) games for same conference different divisions</td>
<td>2</td>
</tr>
<tr>
<td>Lower bound of total number of home (road) games for same conference different divisions</td>
<td>1</td>
</tr>
<tr>
<td>Lower bound of total number of home and road games for same conference different divisions</td>
<td>3</td>
</tr>
<tr>
<td>The total days of a season</td>
<td>65</td>
</tr>
<tr>
<td>The total number of games of each team</td>
<td>30</td>
</tr>
<tr>
<td>The total number of games on Friday and Saturday</td>
<td>4</td>
</tr>
<tr>
<td>Do not arrange games</td>
<td></td>
</tr>
<tr>
<td>To schedule special games on special date</td>
<td></td>
</tr>
<tr>
<td>To schedule road games for i team in a particular period of time</td>
<td></td>
</tr>
<tr>
<td>cannot use the home team identity to play game on same day</td>
<td></td>
</tr>
<tr>
<td>Each team only has 2 games at most in 3 days</td>
<td></td>
</tr>
</tbody>
</table>

This case was solved by calling the commercial optimization package CPLEX 12.4 on a computer with the platform of Windows 7 operating system and Intel(R) Core(TM) i5-4210U CPU @ 1.70GHz 2.40 GHz. The problem includes 9,374 variables and 1,898 constraints. It expensed 10 CPU seconds to obtain the objective value of 1.525. This objective value is the difference of highest opponent competitiveness, Hornets, and the lowest one of Clippers. The other results are shown in Figure 2.

Figure 2. Test results for opponent competitiveness of each team in the verified case

Schedules for all of teams can follow the necessary regulations. Tables 3 and 4 illustrate the schedules of Clippers and Lakers. The number of plays against each opponent is between 2 and 4 games. Each team plays 30 games for the same home and away ones. The dates limited to play are also avoided to arrange any tournament. Conference and division matches are also followed.
Table 3: Test results for clippers schedule in the verified case

<table>
<thead>
<tr>
<th>Day</th>
<th>Home Team</th>
<th>Road Team</th>
<th>Day</th>
<th>Home Team</th>
<th>Road Team</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Wed</td>
<td>Clippers</td>
<td>32</td>
<td>Fri</td>
<td>Clippers</td>
</tr>
<tr>
<td>4</td>
<td>Fri</td>
<td>Clippers</td>
<td>35</td>
<td>Mon</td>
<td>Mavericks</td>
</tr>
<tr>
<td>5</td>
<td>Sat</td>
<td>Hawks</td>
<td>36</td>
<td>Tue</td>
<td>Clippers</td>
</tr>
<tr>
<td>7</td>
<td>Mon</td>
<td>Clippers</td>
<td>38</td>
<td>Thu</td>
<td>Rockets</td>
</tr>
<tr>
<td>9</td>
<td>Wed</td>
<td>Mavericks</td>
<td>40</td>
<td>Sat</td>
<td>Clippers</td>
</tr>
<tr>
<td>11</td>
<td>Fri</td>
<td>Timberwolves</td>
<td>43</td>
<td>Tue</td>
<td>Cavaliers</td>
</tr>
<tr>
<td>12</td>
<td>Sat</td>
<td>Nuggets</td>
<td>48</td>
<td>Sun</td>
<td>Clippers</td>
</tr>
<tr>
<td>14</td>
<td>Mon</td>
<td>Bulls</td>
<td>50</td>
<td>Tue</td>
<td>Timberwolves</td>
</tr>
<tr>
<td>16</td>
<td>Wed</td>
<td>Clippers</td>
<td>53</td>
<td>Fri</td>
<td>Bulls</td>
</tr>
<tr>
<td>18</td>
<td>Fri</td>
<td>Lakers</td>
<td>54</td>
<td>Sat</td>
<td>Clippers</td>
</tr>
<tr>
<td>19</td>
<td>Sat</td>
<td>Nets</td>
<td>57</td>
<td>Tue</td>
<td>Lakers</td>
</tr>
<tr>
<td>26</td>
<td>Sat</td>
<td>Clippers</td>
<td>59</td>
<td>Thu</td>
<td>Celtics</td>
</tr>
<tr>
<td>28</td>
<td>Mon</td>
<td>Clippers</td>
<td>62</td>
<td>Sun</td>
<td>Nuggets</td>
</tr>
<tr>
<td>29</td>
<td>Tue</td>
<td>Clippers</td>
<td>64</td>
<td>Tue</td>
<td>Rockets</td>
</tr>
<tr>
<td>31</td>
<td>Thu</td>
<td>Hornets</td>
<td>65</td>
<td>Wed</td>
<td>Clippers</td>
</tr>
</tbody>
</table>

Table 4: Test results for lakers schedule in the verified case

<table>
<thead>
<tr>
<th>Day</th>
<th>Home Team</th>
<th>Road Team</th>
<th>Day</th>
<th>Home Team</th>
<th>Road Team</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tue</td>
<td>Timberwolves</td>
<td>40</td>
<td>Sat</td>
<td>Timberwolves</td>
</tr>
<tr>
<td>5</td>
<td>Sat</td>
<td>Bulls</td>
<td>42</td>
<td>Mon</td>
<td>Nets</td>
</tr>
<tr>
<td>9</td>
<td>Wed</td>
<td>Celtics</td>
<td>43</td>
<td>Tue</td>
<td>Mavericks</td>
</tr>
<tr>
<td>14</td>
<td>Mon</td>
<td>Cavaliers</td>
<td>46</td>
<td>Fri</td>
<td>Lakers</td>
</tr>
<tr>
<td>15</td>
<td>Tue</td>
<td>Hornets</td>
<td>47</td>
<td>Sat</td>
<td>Nuggets</td>
</tr>
<tr>
<td>17</td>
<td>Thu</td>
<td>Lakers</td>
<td>49</td>
<td>Mon</td>
<td>Lakers</td>
</tr>
<tr>
<td>18</td>
<td>Fri</td>
<td>Lakers</td>
<td>51</td>
<td>Wed</td>
<td>Nuggets</td>
</tr>
<tr>
<td>25</td>
<td>Fri</td>
<td>Lakers</td>
<td>52</td>
<td>Thu</td>
<td>Lakers</td>
</tr>
<tr>
<td>27</td>
<td>Sun</td>
<td>Lakers</td>
<td>54</td>
<td>Sat</td>
<td>Clippers</td>
</tr>
<tr>
<td>28</td>
<td>Mon</td>
<td>Lakers</td>
<td>56</td>
<td>Mon</td>
<td>Lakers</td>
</tr>
<tr>
<td>30</td>
<td>Wed</td>
<td>Lakers</td>
<td>57</td>
<td>Tue</td>
<td>Lakers</td>
</tr>
<tr>
<td>31</td>
<td>Thu</td>
<td>Lakers</td>
<td>60</td>
<td>Fri</td>
<td>Lakers</td>
</tr>
<tr>
<td>33</td>
<td>Sat</td>
<td>Rockets</td>
<td>61</td>
<td>Sat</td>
<td>Lakers</td>
</tr>
<tr>
<td>35</td>
<td>Mon</td>
<td>Lakers</td>
<td>64</td>
<td>Tue</td>
<td>Lakers</td>
</tr>
<tr>
<td>39</td>
<td>Fri</td>
<td>Rockets</td>
<td>65</td>
<td>Wed</td>
<td>Hawks</td>
</tr>
</tbody>
</table>

**Real-World Case**

The proposed model was also applied to solve the real-world case for 2015-2016 regular season. This case has 153,032 variables and 11,866 constraints. The solution time is 2,174 CPU seconds. The highest opponent competitiveness occurs on Timberwolves for 42.298, while the lowest opponent competitiveness appears on Hawks for 39.866. The gap of 2.432 between these two teams is smaller than the difference of opponent competitiveness, 3.711, in 2015-16 season official schedule. The detailed results for opponent competitiveness are shown in Figures 3 and 4. The preliminary numerical experiments revealed that the proposed model can obtain promising results for the tournament arrangement of the NBA.
CONCLUSION AND SUGGESTIONS

Professional sport tournaments are very fascinating. Every league has accumulated a lot of fans and then brought huge benefits. Tournament scheduling problem is an important part for running a professional sport system. This study has proposed a mathematical model to schedule the professional sports tournament schedule for the NBA with a fair perspective. The numerical experiments revealed that this model can obtain promising results.

This model or the formulated perspective can also apply to other sports for tournament scheduling. Although this study took the winning percentages to be the objective, other objective concept, such as travel time, travel distance, back to back quantities, etc., can be also involved in the future research for the NBA.
REFERENCES


