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A tournament scheduling model for National Basketball Association (NBA) regular season games

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Abstract

Aim: This research aims to develop a mathematical programming model for the NBA tournament schedule that considers the equality of the teams that each team will face in the bracket. Professional sports leagues rely heavily on tournaments to maintain and grow their fan bases. However, there are many moving parts to consider when planning a tournament, including the dates, locations, opponents, and more.
Methodology: To level the playing field between teams, an Integer Programming (IP) model was proposed.
Results Numerical experiments showed that this model could improve the NBA's tournament format.
Novelty/Implications: To schedule the NBA's professional sports tournaments in a balanced manner, a mathematical model has been proposed in this study. This model or the developed viewpoint can be used in various sports for tournament schedules.

Key Words: Tournament Scheduling, National Basketball Association (NBA), Integer Programming (IP), Opponent Competitiveness

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INTRODUCTION

Because of the rise of specialized professional athleticism, sports have become more than just a pastime; they are also a lucrative industry. In particular, many people from different parts of the world follow the same professional sports leagues, such as the National Football League (NFL), Major League Baseball (MLB), and the National Basketball Association (NBA) in the United States. Professional athletic competitions can generate massive benefits, including massive viewership for TV broadcasts and sales of tickets and ancillary goods.

One of the most crucial promotional tools for professional leagues is the tournament schedule. The tournament pairings of their favorite athletes are always a focal point for spectators. Because the schedules affect the overall performance of all teams and all athletes, tournament toughness and fairness are major topics of discussion among team management, athletes, and fans. However, scheduling tournaments for a large league like the NBA with 30 teams is not a simple task by manual arrangement due to the numerous and complex restrictions that must be adhered to. Moreover, the goals of strictness and equity in such regulations would not be achieved. With this in mind, we propose a tournament scheduling model for NBA regular season games with a fair objective. Plays can appeal more to fans, and larger crowds can be attracted with the right scheduling. In addition, this setup has the potential to improve athletes' performances while mitigating their adverse psychological and physiological effects.

Basketball arenas for NBA teams can be found in various American and Canadian cities. There are several conferences and divisions within them. Every team has the same number of home and away games spread out over a set schedule of days during the season. When teams from different conferences and divisions compete against one another, the game's rules can be interpreted in various ways. Home and away venues, as well as available dates, commercial considerations, and the absence of conflicting events in the venues, are the deciding factors. This research aims to develop a mathematical programming model for NBA regular season game scheduling in

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tournaments. An IP model was proposed to reduce the disparity in opponent competitiveness among all teams, subject to the scheduling rules. This research checks the results of the solutions in a few specific cases to ensure they can be used in real-world programming. In Section 2 of this paper, the remaining content reviews the literature on the topic of sports tournament scheduling. The NBA tournament schedule rules and the proposed model are presented in Section 3. The outcomes of various test cases are discussed in Section 4. In the final section, our study is summed up, and some further study directions are suggested.

LITERATURE REVIEW

Scheduling problems in sports are required such as the arrangement for the tournaments and umpires. Wright (2009) reviewed the works using Operations Research (OR) techniques for about 50 years. Ribeiro (2012) analyzed the fundamental problems for sport scheduling and provided a survey of applications of optimization methods to scheduling problems in professional leagues of different sport disciplines. The main problems arising in tournament scheduling include issues such as breaks minimization, distance minimization, the Traveling Tournament Problem (TTP), and carry-over effects minimization. These problems have been solved by different exact and approximate approaches, including integer programming, constraint programming, metaheuristics, and hybrid methods. Some efforts are devoted to the specific problems. Costa (1995) combined the mechanisms of genetic algorithms and tabu search to solve National Hockey League (NHL) optimization problems, hoping to reduce costs. This hybrid method developed is well suited for Open Shop Scheduling Problems. Dilkina and Havens (2004) used Constraint Satisfaction Problem (CSP) to schedule 256 games in 17 weeks for 32 teams of the NFL. These teams were divided into the National Football Conference (NFC) and the American Football Conference (AFC). Some home/away spacing constraints and other related regulations required to be adhered. An available schedule had to be used with the US television network live. They provided an overview of the constraint solving methodology employed and the implementation of the NFL prototype system. Duran, Guajardo, and Wolf-Yadlin (2012) proposed an integer linear programming (ILP) model to schedule the Second Division of the Chilean professional soccer league. Geographical restrictions are particularly important because of the special geographical environment of Chile for long bus travel distances. Chilean league officials have successfully used this model to schedule all five Second Division tournaments between 2007 and 2010.

There were studies focused on the nonprofessional basketball games. Nemhauser and Trick (1998) developed a combination of integer programming and enumerative techniques to schedule nine universities in the Atlantic Coast Conference (ACC). A basketball competition assigned schools to play home and road games against each other over a nine-week period. This work was applied to make reasonable schedules in 1997-1998 seasons. Wright (2006) used a variant of simulated Annealing (SA) to schedule games for National Basketball League of New Zealand. This approach was applied to make the schedule in 2004 season.

For the NBA, recent research devoted to various perspectives, such as statistical analysis for winning playoffs (Summers 2013), identifying basketball performance indicators in regular season and playoff games (Garcia et al. 2013), time trends for injuries and illness, and their relation to performance (Podlog et al. 2015). The studies for tournament scheduling are relatively limited. Bean and Birge (1980) constructed schedules of NBA based on Traveling Salesman Problem (TSP). The objective is to reduce the total number of passenger miles for the league travels, 22 teams in the NBA at the time. This approach successfully developed schedules for the 1978-1979 and 1979-1980 seasons with savings of 20.4% costs. Actually, current structure in the NBA has been changed than 20 years ago. More teams and more fans make the tournament scheduling problem become more complicated and commercialized.

METHODOLOGY

This section briefly introduces current teams in the NBA and scheduling rules for their tournaments. A proposed mathematical model can then be formulated with the objective of minimizing the difference of opponent competitiveness among all teams. Opponent competitiveness is defined as the sums of opponent winning percentages in the last season for every game.

Problem Description

The NBA has been developed for nearly 70 years. Initially, there are only 11 teams in NBA, each team playing 60 games in a season. Currently, the NBA has become a huge sport league, with 30 teams homed at different cities in USA and Canada. Figure 1 shows the location of home grounds for these teams. They are divided into 2 conferences, eastern conference and western conference, and belong to 6 divisions, respectively.

Eastern Conference

a. Atlantic division includes Boston Celtics, Brooklyn Nets, New York Knicks, Philadelphia 76ers, and Toronto Raptors.

b. Central division includes Chicago Bulls, Cleveland Cavaliers, Detroit Pistons, Indiana Pacers, and Milwaukee Bucks.

c. Southeast division includes Atlanta Hawks, Charlotte Hornets, Miami Heat, Orlando Magic and Washington Wizards.

Western Conference

a. Southwest division includes Dallas Mavericks, Houston Rockets, Memphis Grizzlies, New Orleans Pelicans, and San Antonio Spurs.

b. Northwest division includes Denver Nuggets, Minnesota Timberwolves, Oklahoma City Thunder, Portland Trail Blazers, and Utah Jazz.

c. Pacific division includes Golden State Warriors, Los Angeles Clippers, Los Angeles Lakers, Phoenix Suns, and Sacramento Kings.

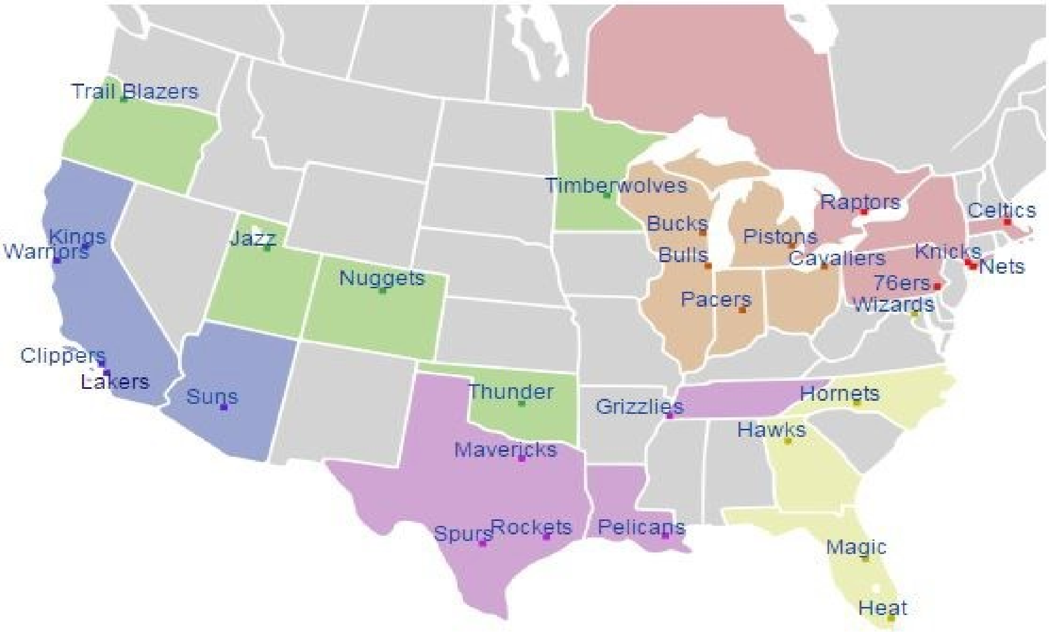

Figure 1. Location of home grounds for all NBA teams

Related scheduling regulations in scheduling tournaments in the NBA are collected as follows:

- Each team must play 82 games in a season, 41 home games and 41 road games.
- Each team only has 1 game at most in a day.
- Each team only has 2 games at most within 3 days.
- More teams have to play on Friday and Saturday.
- Lakers and Clippers use the same court, i.e., Staples Center, so they cannot use the home team identity to play game on same day.
- No games can be played at the Thanksgiving Day, Christmas Eve, All-Star Game and NCAA finals.
- A team has to play road games when its home court cannot be used in a particular period of time.

- Each team has to play 4 games (2 home games and 2 road games) with the other 4 teams in the same division.
- Each team has to play 2 games (1 home game and 1 road game) with the other 15 teams in different conferences.
- Each team has to play 3 or 4 games (1 home game and 2 road game, 2 home game and 1 road game, or 2 home game and 2 road game) with the other 10 teams in the same conference but different divisions.
- Some special games are arranged on special dates, such as a deliberate game for Warriors and Cavaliers (last season's finals) at Christmas in 2015-16 season.

Mathematical Model

The proposed model aims to minimize the difference of opponent competitiveness among all teams subject to the scheduling regulations mentioned in the methodology Section. Before introducing the contents of the model, notation for this model is defined as follows:

Indices:

d: Indices for scheduling days

i, j: Indices for team

Sets:

$\bar{W}$: the set of days for Friday and Saturday.

$\bar{H}$: the set of days that cannot arrange games.

$\bar{R}_i$: the set of days that have to arrange road games for team i.

$\bar{S}$: the set of days that have to arrange particular games.

E_d: the teams must play on day d in.

X : the set of teams for shared home.

Decision variables:

x_{ij}^d: if team i plays home game with team j at day d ground; 1 for yes, 0 otherwise.

Z_i : opponent competitiveness for team i, i.e., the sum of opponent winning percentages in the last season for every game for team i.

Z_h : the highest opponent competitiveness among all teams.

Z_l : the lowest opponent competitiveness among all teams.

Parameters:

C_i : the conference of team i.

V_i : the division of team i.

W_j: the last seasonal winning percentage of team j.

D: the total days of a regular season.

K: the total number of teams.

a: the total number of games for each team playing in a regular season.

f: the total number of home (away) games with teams of the same conference and the same division.

g: the total number of home (away) games with teams of the different conference.

h: upper bound of total number of home (away) games with teams of the same conference but different divisions.

l: lower bound of total number of home (away) games with teams of the same conference but different divisions.

m: maximum days for consecutive plays.

n: upper bound of the consecutive games in maximum days for consecutive plays.

q: lower bound of total number of home and road games with teams of the same conference but different divisions.

r: a ratio of teams playing on Friday and Saturday.

As the Equation (1), the objective function of this model is to minimize the difference of opponent competitiveness. It can be calculated by the difference of Z_h and Z_l.

$$Minimize = Z_h - Z_l \tag{1}$$

Some constraints require to be followed. Opponent competitiveness definition constraints include Equation (2) to (4). Equation (2) is the sum of opponent winning percentages in the last season for every game for team i.

$$Z_i = \sum_d \sum_j w_j x_{ij}^d + \sum_d \sum_j w_j x_{ji}^d \qquad \forall i \tag{2}$$

$$Z_h \geq Z_i \qquad \forall i \tag{3}$$

$$Z_l \leq Z_i \qquad \forall i \tag{4}$$

Equation (5) to (7) are the limitations for games. Equation (5) means that the total number of home games must be equal to road games. Equation (6) enforces that the total number of games of each team must be the same. Equation (7) limits each team to play only one game at most in a day.

$$\sum_d \sum_j x_{ij}^d = \sum_d \sum_j x_{ji}^d \qquad \forall i \tag{5}$$

$$\sum_d \sum_j x_{ij}^d + \sum_d \sum_j x_{ji}^d = a \qquad \forall i \tag{6}$$

$$\sum_j x_{ij}^d + \sum_j x_{ji}^d \leq 1 \qquad \forall i, d \tag{7}$$

Equation (8) can limit each team to play only n games at most in consecutive m days.

$$\sum_{d'=d}^{d+(m-1)} \sum_j x_{ij}^d + \sum_{d'=d}^{d+(m-1)} \sum_j x_{ji}^d \leq n \qquad \forall i, d = 1, 2, ..., D - (m+1) \tag{8}$$

If two or more teams use same court, they cannot use the home team identity to play game on the same day as Equation (9).

$$\sum_j x_{ij}^d + \sum_j x_{i'j}^d \leq 1 \qquad \forall d, (i, i')\varepsilon X \tag{9}$$

Equation (10) enforces that specific days can play no games. On the other hand, certain Fridays and Saturdays must play games for commercial considerations as Equation (11). Equation (12) can schedule special games on special dates for certain terms. Equation (13) limits a team to play road games when its home court cannot be used in a particular period of time.

$$\sum_i \sum_j x_{ij}^d + \sum_i \sum_j x_{ji}^d = 0 \qquad \forall d\varepsilon \bar{H} \tag{10}$$

$$\sum_i \sum_j x_{ij}^d \geq \frac{rK}{2} \qquad \forall d\varepsilon \bar{W} - \bar{H} \tag{11}$$

$$x_{ij}^d + x_{ji}^d = 1 \qquad \forall d\varepsilon \bar{S}, (i, j)\varepsilon E_d \tag{12}$$

$$\sum_j x_{ij}^d = 0 \qquad \forall i, d\varepsilon \bar{R}_i \tag{13}$$

Equations (14) to (22) are constraints playing with opponents in conferences and divisions. Equations (14) and (15) consider the total number of home (away) games for same conference same division. Equations (16) to (17) limit the total number of home (away) games for different conferences. Equations (18) to (22) limit the total number of home (away) games for same conference but different divisions. Equations (23) and (24) are nature of the variables.

$$\sum_d x_{ij}^d = f \qquad \forall i, j \neq i (V_i = V_j) \tag{14}$$

$$\sum_{d} x_{ji}^{d} = f \qquad \forall i, j \neq i (V_i = V_j) \tag{15}$$

$$\sum_{d} x_{ij}^{d} = g \qquad \forall i, j \neq i (C_i \neq C_j) \tag{16}$$

$$\sum_{d} x_{ji}^{d} = g \qquad \forall i, j \neq i (C_i \neq C_j) \tag{17}$$

$$\sum_{d} x_{ij}^{d} + \sum_{d} x_{ji}^{d} \geq q \qquad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \tag{18}$$

$$\sum_{d} x_{ij}^{d} \leq h \qquad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \tag{19}$$

$$\sum_{d} x_{ij}^{d} \geq l \qquad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \tag{20}$$

$$\sum_{d} x_{ji}^{d} \leq h \qquad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \tag{21}$$

$$\sum_{d} x_{ji}^{d} \geq l \qquad \forall i, j \neq i (C_i = C_j, V_i \neq V_j) \tag{22}$$

$$x_{ij}^{d} \varepsilon \{0, 1\} \tag{23}$$

$$Z_h, Z_l, Z_i \geq 0 \tag{24}$$

RESULTS AND DISCUSSION

The proposed model is an integer programming problem. This section presents a verified case with smaller problem scale for discussion in detail. Then, the solution results for the real-world case of 2015-2016 regular season tournaments are reported.

Verified Case

The test case only selects 12 teams, as shown in Table 1, for tournaments scheduling to play 30 games each team within 65 days. Detailed settings of input data for scheduling regulations are presented in Table 2.

Table 1: Teams in the verified case

Team	Location	Winning Percentages (last season)	Conference	Division
Celtics	Boston	0.488	Eastern	Atlantic
Nets	Brooklyn	0.463	Eastern	Atlantic
Bulls	Chicago	0.610	Eastern	Central
Cavaliers	Cleveland	0.646	Eastern	Central
Hawks	Atlanta	0.732	Eastern	Southeast
Hornets	Charlotte	0.402	Eastern	Southeast
Mavericks	Dallas	0.610	Western	Southwest
Rockets	Houston	0.683	Western	Southwest
Nuggets	Denver	0.366	Western	Northwest
Timberwolves	Minnesota	0.195	Western	Northwest
Clippers	Los Angeles	0.683	Western	Pacific
Lakers	Los Angeles	0.256	Western	Pacific

Table 2: Settings of input data for the test case

Regulations	Settings
The total number of home (road) games for same conference same division	2
The total number of home (road) games for different conferences	1
Upper bound of total number of home (road) games for same conference different divisions	2
Lower bound of total number of home (road) games for same conference different divisions	1
Lower bound of total number of home and road games for same conference different divisions	3
The total days of a season	65
The total number of games of each team	30
The total number of games on Friday and Saturday	4
Do not arrange games	Day 20, 21, 22, 23, 24
To schedule special games on special date	Day 2 Celtics vs. Nets Day 14 Cavaliers vs. Lakers Day 14 Bulls vs. Clippers
To schedule road games for i team in a particular period of time	Clippers Day 10, 11, 12, 13, 14 Lakers Day 10, 11, 12, 13, 14
cannot use the home team identity to play game on same day	Lakers and Clippers
Each team only has 2 games at most in 3 days	

This case was solved by calling the commercial optimization package CPLEX 12.4 on a computer with the platform of Windows 7 operating system and Intel(R) Core(TM) i5-4210U CPU @ 1.70GHz 2.40 GHz. The problem includes 9,374 variables and 1,898 constraints. It expensed 10 CPU seconds to obtain the objective value of 1.525. This objective value is the difference of highest opponent competitiveness, Hornets, and the lowest one of Clippers. The other results are shown in Figure 2.

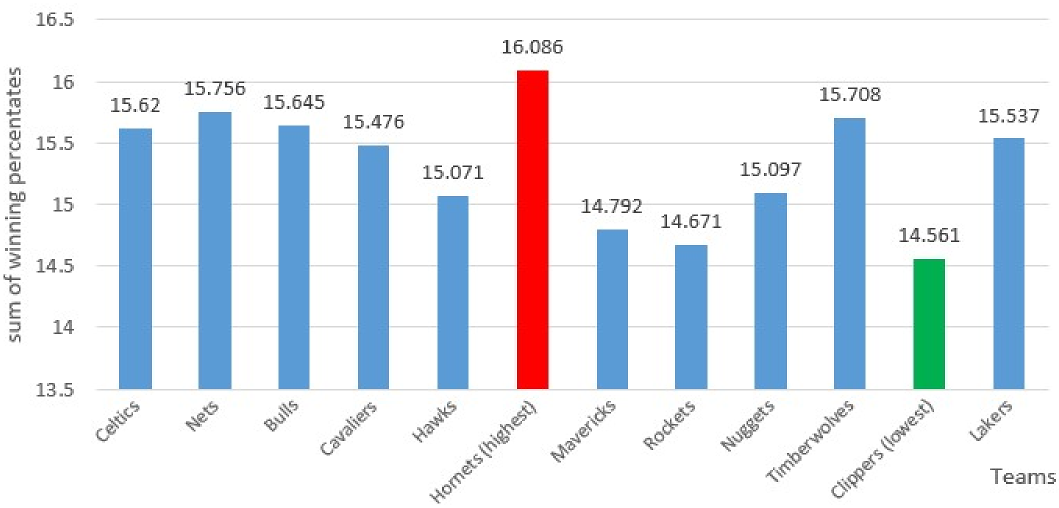

Figure 2. Test results for opponent competitiveness of each team in the verified case

Schedules for all of teams can follow the necessary regulations. Tables 3 and 4 illustrate the schedules of Clippers and Lakers. The number of plays against each opponent is between 2 and 4 games. Each team plays 30 games for the same home and away ones. The dates limited to play are also avoided to arrange any tournament. Conference and division matches are also followed.

Table 3: Test results for clippers schedule in the verified case

Day		Home Team	Road Team	Day		Home Team	Road Team
2	Wed	Clippers	Rockets	32	Fri	Clippers	Nuggets
4	Fri	Clippers	Hawks	35	Mon	Mavericks	Clippers
5	Sat	Hawks	Clippers	36	Tue	Clippers	Nets
7	Mon	Clippers	Mavericks	38	Thu	Rockets	Clippers
9	Wed	Mavericks	Clippers	40	Sat	Clippers	Cavaliers
11	Fri	Timberwolves	Clippers	43	Tue	Cavaliers	Clippers
12	Sat	Nuggets	Clippers	48	Sun	Clippers	Hornets
14	Mon	Bulls	Clippers	50	Tue	Clippers	Timberwolves
16	Wed	Clippers	Mavericks	53	Fri	Clippers	Bulls
18	Fri	Lakers	Clippers	54	Sat	Clippers	Lakers
19	Sat	Nets	Clippers	57	Tue	Lakers	Clippers
26	Sat	Clippers	Timberwolves	59	Thu	Celtics	Clippers
28	Mon	Clippers	Lakers	62	Sun	Nuggets	Clippers
29	Tue	Clippers	Rockets	64	Tue	Rockets	Clippers
31	Thu	Hornets	Clippers	65	Wed	Clippers	Celtics

Table 4: Test results for lakers schedule in the verified case

Day		Home Team	Road Team	Day		Home Team	Road Team
1	Tue	Timberwolves	Lakers	40	Sat	Timberwolves	Lakers
5	Sat	Bulls	Lakers	42	Mon	Nets	Lakers
9	Wed	Celtics	Lakers	43	Tue	Mavericks	Lakers
14	Mon	Cavaliers	Lakers	46	Fri	Lakers	Nuggets
15	Tue	Hornets	Lakers	47	Sat	Nuggets	Lakers
17	Thu	Lakers	Celtics	49	Mon	Lakers	Timberwolves
18	Fri	Lakers	Clippers	51	Wed	Nuggets	Lakers
25	Fri	Lakers	Hornets	52	Thu	Lakers	Timberwolves
27	Sun	Lakers	Nets	54	Sat	Clippers	Lakers
28	Mon	Clippers	Lakers	56	Mon	Lakers	Mavericks
30	Wed	Lakers	Hawks	57	Tue	Lakers	Clippers
31	Thu	Lakers	Nuggets	60	Fri	Lakers	Rockets
33	Sat	Rockets	Lakers	61	Sat	Lakers	Mavericks
35	Mon	Lakers	Bulls	64	Tue	Lakers	Cavaliers
39	Fri	Rockets	Lakers	65	Wed	Hawks	Lakers

Real-World Case

The proposed model was also applied to solve the real-world case for 2015-2016 regular season. This case has 153,032 variables and 11,866 constraints. The solution time is 2,174 CPU seconds. The highest opponent competitiveness occurs on Timberwolves for 42.298, while the lowest opponent competitiveness appears on Hawks for 39.866. The gap of 2.432 between these two teams is smaller than the difference of opponent competitiveness, 3.711, in 2015-16 season official schedule. The detailed results for opponent competitiveness are shown in Figures 3 and 4. The preliminary numerical experiments revealed that the proposed model can obtain promising results for the tournament arrangement of the NBA.

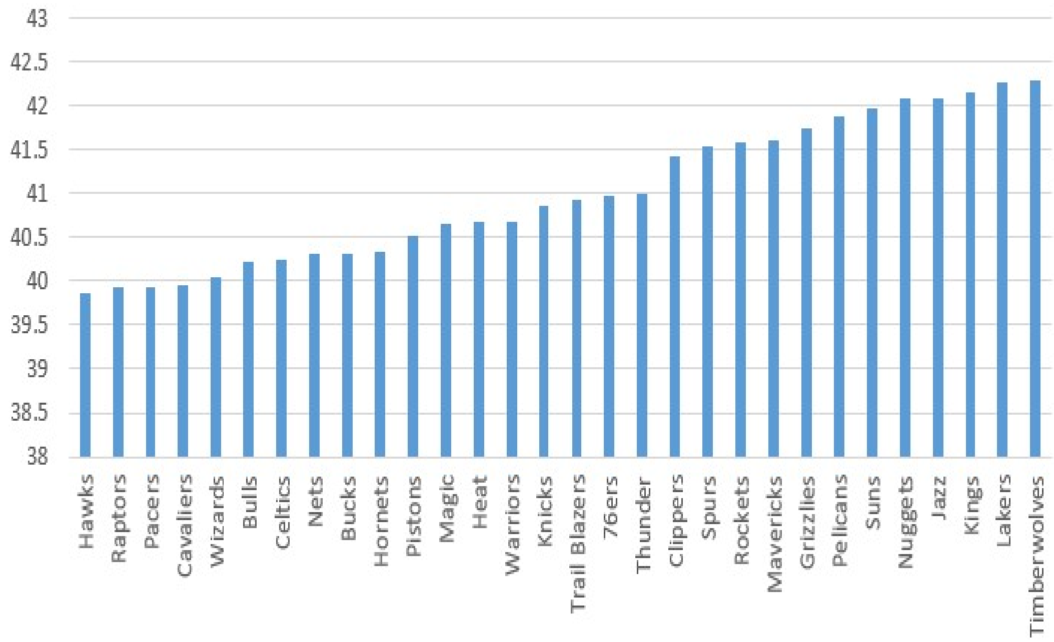

Figure 3. Opponent competitiveness of each team in solution results

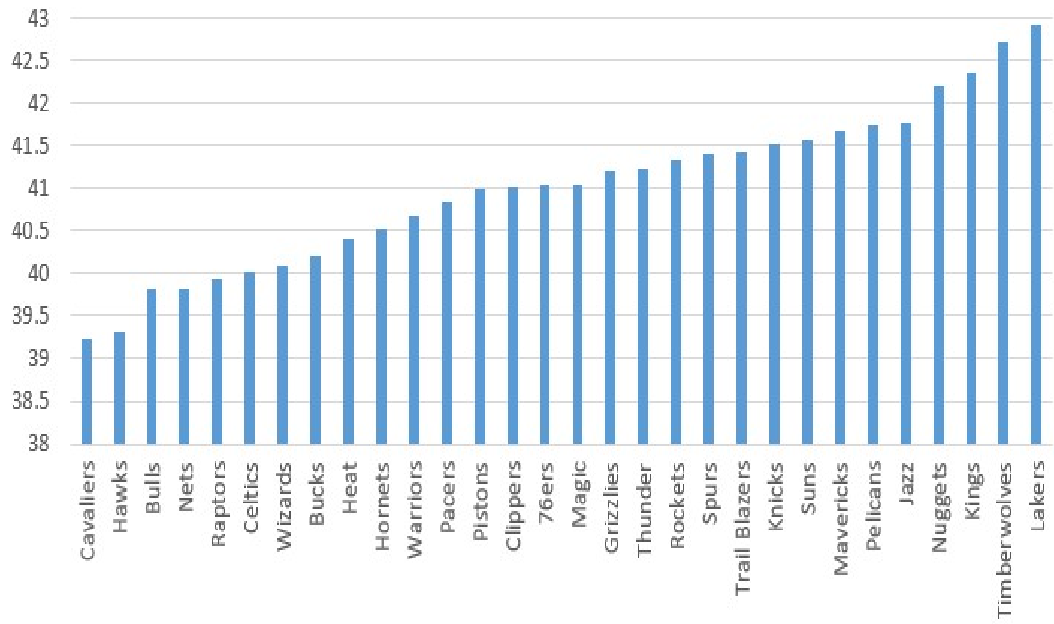

Figure 4. Opponent competitiveness of each team in 2015-16 season official schedule

CONCLUSION, RECOMMENDATIONS AND IMPLICATIONS

Professional sports have become a business market with high rewards in global economies. In particular, the high-level leagues can attract a group of sport populations over global countries, such as the National Basketball Association (NBA). The NBA currently consists of 30 teams homed in different cities in the USA and Canada. The whole tournament of its regular season every year lasts to play nearly 24 weeks. Nevertheless, the tournament scheduling problem is complicated and large-scale and subject to many factors, such as dates, venues, opponents of games, etc. The tournament scheduling problem is an important part of running a professional sports system. This study has proposed a mathematical model to schedule the professional sports tournament schedule for the NBA with a fair perspective. The numerical experiments revealed that this model can obtain promising results.

Limitations and Future Directions

This investigation and the proposed model is only limited to NBA. Nevertheless, this model or the formulated perspective can also apply to other sports for tournament scheduling. Although this study took the winning percentages to be the objective, other objective concepts, such as travel time, travel distance, back to back quantities, etc., can be also involved in future research for the NBA.

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